# Neutrino masses and mixing angles: a tribute to Guido Altarelli

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INVISIBLES JOURNAL CLUB

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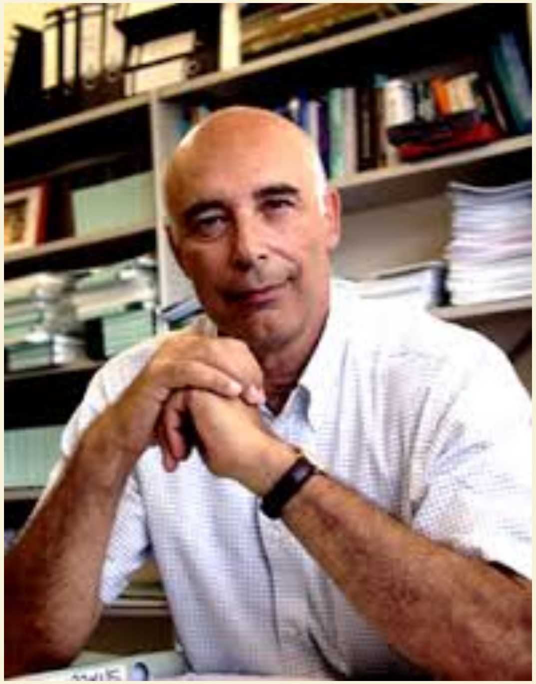


Guido Altarelli [1941-2015]:
a true giant of particle physics.
His contributions to physics span all subjects, from strong to electroweak interactions, from neutrinos to theories beyond the Standard Model.

His best known contribution is the derivation of the QCD evolution equations for parton densities (1977) known as the Altarelli-Parisi or DGLAP equations.

Here:
his contribution to the field of neutrino masses and mixing angles to testify the wideness of his interests

- member of the Polish Academy of Sciences
- 2011 Julius Wess Award
- 2012 J. J. Sakurai Prize for Theoretical Particle Physics [APS]
- 2015 High Energy and Particle Physics Prize - EPS HEPP Prize

# Plan of the talk

1969 – 1997:
-- neutrino timeline

1998 – 2005:
-- struggling with textures
-- abelian flavour symmetries
-- GUTs

2005 -2011:
-- discrete flavour symmetries

2011 -2013:
-- new directions

1998: convincing evidence of neutrino oscillations [SuperKamiokande]

2002: solar neutrino problem solved [SNO CC and NC, Kamland]

2011: T2K, Minos, Daya Bay, RENO measure $\vartheta_{13}$

# Solar Neutrino Timeline

| | |
|---|---|
| 1969 | 1st detection of solar neutrinos by R. Davis at the Homestake mine $\nu_e + {}^{37}Cl \rightarrow e^- + {}^{37}Ar$ solar ν problem starts, no other solar ν experiments for 20 yr! |
| 1969 | solution in terms of $\nu_e \rightarrow \nu_\mu$ oscillations by Gribov and Pontecorvo |
| 1974 | GUT proposed by Georgi and Glashow |
| 1977 | see-saw mechanism for neutrino masses [Minkowski, Gell-Mann, Ramond, Slanski and Yanagida] |
| 1978 – 1986 | Wolfenstein, Mikheyev, Smirnov (MSW effect) sizeable solar $\nu_e$ conversion possible with small mixing angle |
| 1987 | detection of neutrinos from SN1987A by Kamiokande, IMB, Baksan. Kamiokande lower the E threshold below solar ν energies ~ 10 MeV |
| 1989 | $N_\nu = 3$ from LEP |
| 90s | SAGE, GALLEX, GNO $\nu_e + {}^{71}Ga \rightarrow e^- + {}^{71}Ge$ confirm the solar ν problem in the low-energy region of ν spectrum |
| 1994 | $m_{\nu_e} < 2.2\ eV$ [Troitsk] |

# Atmospheric Neutrino Timeline

1978 first measurement of

$$\Phi_{th}(\nu_\mu)/\Phi_{\exp}(\nu_\mu) = 1.6 \pm 0.4$$

Crouch, M.F., Landecker, P.B., Lathrop, J.F., Reines, F., Sandie, W.G., Sobel, H.W. *et al.* (1978) Cosmic-ray muon fluxes deep underground: Intensity vs depth, and the neutrino-induced component. Phys. Rev. D **18**, 2239–2252.

80s several proton decay experiments started M = 100 – 3000 tons
atmospheric ν, serious background for p-decay searches, are carefully studied
Kamiokande, IMB, Soudan

$$R = (\mu/e)_{data} / (\mu/e)_{MC} \approx 0.6$$

atmospheric ν problem

## Prejudices < 1997

solar ν problem:
several solutions possible
-- SSM not correct
-- resonant spin-flavour precession of ν
-- FCNC solution
-- MSW SA attractive

atmospheric ν problem:
it will fade away since it requires a large mixing angle

One can in principle explain the data if one assumes neutrino oscillations,

However, at that time, it was commonly believed that the mixing angles between neutrinos must be small, since the corresponding mixing angles between the quarks are known to be small. Therefore, the result and the oscillation interpretation were not accepted by physicists, since they implied that the mixing angle between neutrinos is large.

[T. Kajita 2010]

# 1997 - 1998 turnpoint

1997 solar sound speed from helioseismology compared with predictions of SSM (test T-profile in solar interior)

➡ SSM reliable

Bahcall, Pinsonneault, Sarbani Basu, Christensen-Dalsgaard
Phys.Rev.Lett. 78 (1997) 171

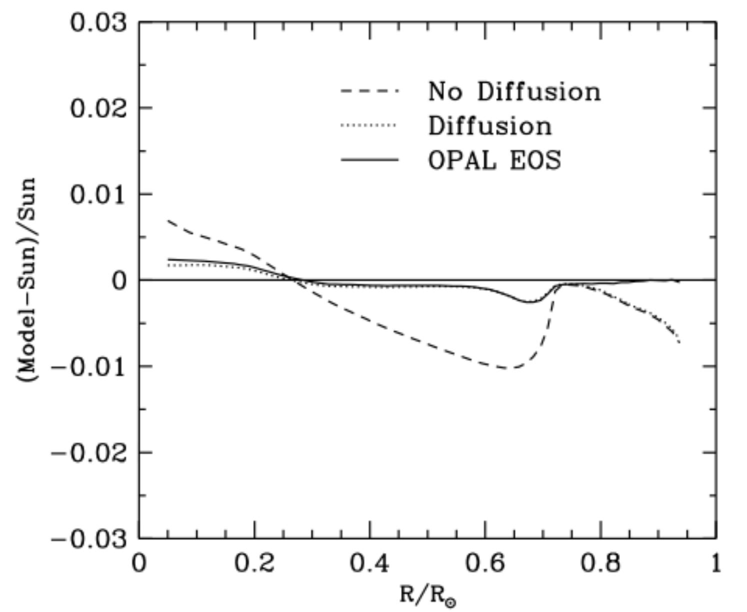



FIG. 1. Comparison of sound speeds predicted by different standard solar models with the sound speeds measured by helioseismology. There are no free parameters in the models;

1996 Superkamiokande starts, atmospheric ν data shown at Neutrino '98

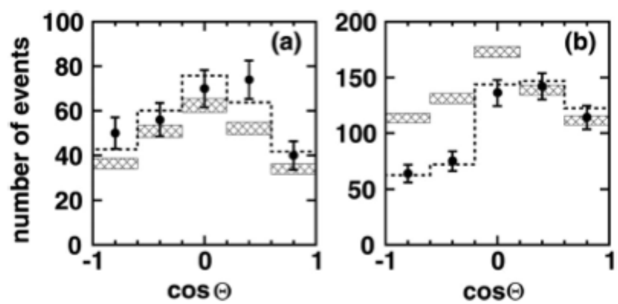



Fig. 14. Zenith angle distributions for multi-GeV atmospheric neutrino events reported at the Nuetrino'98 conference based on 535 days exposure of the Super-Kamiokande detector. The left and right panels show the distributions for $e$-like and $\mu$-like events, respectively. Θ shows the zenith angle, and $\cos\Theta = 1$ and $-1$ represent events whose direction is vertically downward-going and upward-going, respectively.

⬇

-- zenith angular distributions of atmospheric ν

-- oscillation solution becomes compelling

-- determination of

$$(\Delta m^2_{atm}, \sin^2 2\vartheta_{23})$$

≈1 -> maximal mixing

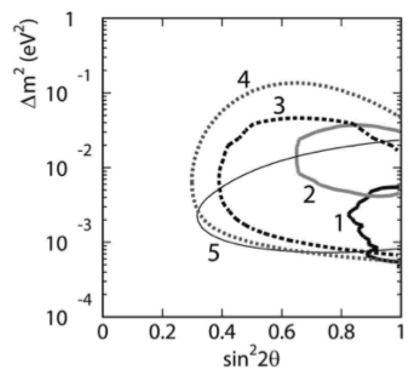



Fig. 15. Allowed parameter regions of $\nu_\mu \to \nu_\tau$ oscillations from Super-Kamiokande and Kamiokande shown at the Neutrino'98 conference.[7] Contours are obtained based on; (1) contained events from Super-Kamiokande, (2) contained events from Kamiokande, (3) upward through-going events from Super-Kamiokande, (4) upward through-going events from Kamiokande and (5) stop/through ratio analysis for upward-going muons from Super-Kamiokande.

in 1997-98 I was visiting CERN … and Guido took me into the ν world

# Guido "principles" about neutrinos

## a new insight into the flavour puzzle?

Quark sector reasonably well-known at the time, but baseline model for quark masses and mixing angles missing.

neutrino masses and large $\vartheta_{23}$ were interesting new inputs

[Parodi, Rudeau, Stocchi 9802289]

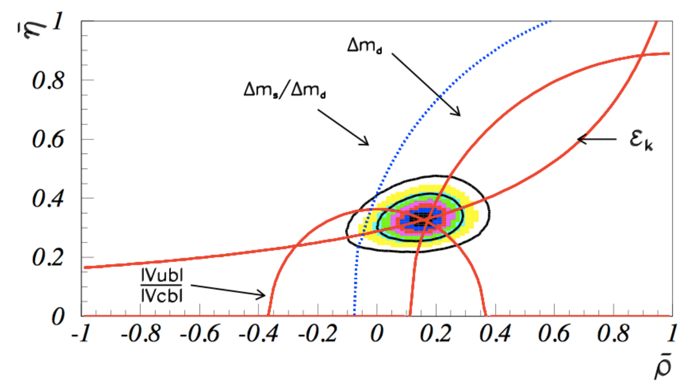



Figure 1: *The allowed region for $\overline{\rho}$ and $\overline{\eta}$ using the parameters listed in Table 1. The contours at 68 % and 95 % are shown. The full lines correspond to the central values of the constraints given by the measurements of $\frac{|V_{ub}|}{|V_{cb}|}$, $|\epsilon_K|$ and $\Delta m_d$. The dotted curve corresponds to the 95 % C.L. upper limit obtained from the experimental limit on $\Delta m_s$.*

## violation of L at a large scale M

" Given that neutrino masses are certainly extremely small, it is really difficult from the theory point of view to avoid the conclusion that L conservation must be violated. In fact, in terms of lepton number violation the smallness of neutrino masses can be explained as inversely proportional to the very large scale where L is violated, of order $M_{GUT}$ or even $M_{Pl}$. "

$$m_\nu \approx \sqrt{\Delta m^2_{atm}} \approx \frac{(\text{EWscale})^2}{M} \quad \Rightarrow \quad M \approx 10^{15}\ GeV$$

"the most impressive numerology that comes out from neutrinos"

[GA, Neutrino 2004, Paris]

## neutrino masses and GUTs

$$m_\nu \approx \frac{(\text{EWscale})^2}{M}$$

very plausible that this arises from the see-saw mechanism

the simplest realization (type I) needs a right-handed neutrino $\nu^c$

" We consider that the existence of RH neutrinos $\nu^c$ is quite plausible because all GUT groups larger than SU(5) require them. In particular the fact that $\nu^c$ completes the representation 16 of SO(10): $16=\bar{5}+10+1$, so that all fermions of each family are contained in a single representation of the unifying group, is too impressive not to be significant. "

"GUTs are the most attractive conjecture for the large scale picture of particle physics. GUT is not the SM, is beyond the SM, but is the most standard physics beyond the SM. Most of us think that there should be something like a GUT."

[GA, Neutrino 2004, Paris]

$$m_\nu = -m_D^{\nu\,T} M^{-1} m_D^\nu$$

$$m_e, m_u, m_d$$

⇒ neutrino masses potentially related to the other charged fermion masses in a GUT

"another big plus of neutrinos is the elegant picture of baryogenesis through leptogenesis (after LEP has disfavoured BG ath the weak scale)"

# The work starts: textures

$$m_\nu = U m_{diag} U^T$$

in the flavour basis

$$U_{fi} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} c & -s & 0 \\ s & c & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

neglecting $\Delta m^2_{sol}$ and $\vartheta_{13}$ and taking $\vartheta_{12}=\pi/4$ or 0

if see-saw, degeneracy need conspiracy between $m_D^\nu$ and M.
$m_\nu$ is quadratic in $m_D^\nu$, any hierachy in $m_D^\nu$ gets amplified in $m_\nu$

| | | $m_{diag}$ | double maximal mixing | single maximal mixing |
|---|---|---|---|---|
| A=NH | A | Diag[0,0,1] | $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1/2 & -1/2 \\ 0 & -1/2 & 1/2 \end{bmatrix}$ | $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1/2 & -1/2 \\ 0 & -1/2 & 1/2 \end{bmatrix}$ |
| B=IH | B1 | Diag[1,-1,0] | $\begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & 0 & 0 \end{bmatrix}$ | $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1/2 & -1/2 \\ 0 & -1/2 & -1/2 \end{bmatrix}$ |
| | B2 | Diag[1,1,0] | $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \end{bmatrix}$ | $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \end{bmatrix}$ |
| C=degenerate | C0 | Diag[1,1,1] | $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ | $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ |
| | C1 | Diag[-1,1,1] | $\begin{bmatrix} 0 & -1/\sqrt{2} & -1/\sqrt{2} \\ -1/\sqrt{2} & 1/2 & -1/2 \\ -1/\sqrt{2} & -1/2 & 1/2 \end{bmatrix}$ | $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ |
| | C2 | Diag[1,-1,1] | $\begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/2 & -1/2 \\ 1/\sqrt{2} & -1/2 & 1/2 \end{bmatrix}$ | $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}$ |
| | C3 | Diag[1,1,-1] | $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ | $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ |

# Guido's favorite texture

$$m_\nu \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & x^2 & x \\ 0 & x & 1 \end{pmatrix} m$$

large mixing requires degenerate states?

$$m_3 = (1 + x^2)m \qquad m_{1,2} = 0$$

here x=O(1) implies large mixing and det[23]=0 guarantees the large splitting needed by atm ν

$$\Delta m^2_{atm} = m^2(1+x^2)^2 \qquad \sin^2 2\vartheta_{23} = \frac{4x^2}{(1+x^2)^2}$$

$$\vartheta_{13} = 0$$

$\Delta m^2_{sol} = 0$ $\qquad \vartheta_{12}$ undetermined

$$\sin^2 2\vartheta_{23} \geq 0.9 \quad [2000]$$

$$0.7 \leq |x| \leq 1.4$$

- compatible with MSW SA, LA LOW and VO

- when embedded in SU(5), compatible with small quark mixing angles

assumptions
-- minimal SU(5) field content (3 light neutrinos)
-- Dirac masses of u,d,e, ν dominated by third generation [LO]

$$\bar{5} = (l, d^c)$$

$$10 = (q, u^c, e^c)$$

$$\Phi_5 = (\Phi_D, \Phi_T)$$

$$\bar{\Phi}_5 = (\bar{\Phi}_D, \bar{\Phi}_T)$$

# fermion masses in minimal SU(5)

$$10\, y_u 10\, \Phi_5 \xrightarrow{10} y_u^{diag}$$

$$\bar{5}\, \frac{w}{M}\, \bar{5}\, \Phi_5 \Phi_5 \xrightarrow{\bar{5}} m_\nu^{diag}$$

$$\bar{5}\, y_d 10\, \bar{\Phi}_5 \longrightarrow y_e = y_d^T$$

must be corrected for 1st and 2nd generations, but OK at the LO

$$(d^c\ l)\, y_d \begin{pmatrix} q \\ e^c \end{pmatrix} \bar{\Phi}_5$$

contains both $V_{CKM}$ and $U_{PMNS}$

| | | | | |
|---|---|---|---|---|
| LEFT | q mixing | <-> | RIGHT | e mixing |
| RIGHT | q mixing | <-> | LEFT | e mixing |

$V_{CKM} \approx 1$ -> small LEFT quark mixing

RIGHT quark mixing completely free [not measurable in weak interactions]

non-hermitian $y_d$

$$y_d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & x \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow \begin{cases} y_d^+ y_d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1+x^2 \end{pmatrix} \\ y_d y_d^+ = \begin{pmatrix} 0 & 0 & 0 \\ 0 & x^2 & x \\ 0 & x & 1 \end{pmatrix} \end{cases}$$

$$V_{CKM} = 1 \times U(\vartheta_C)$$

$$U_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \times U(\vartheta_{12})$$

[Hagiwara, Okamura '98; Berezhiani, Rossi '98 Altarelli, F. '98]

- for a long time prejudice was in favour of hermitian textures $y_{u,d}$ because they were predictive:
  -- Gatto Sartori Tonin relation $\sin\vartheta_C \approx \sqrt{\frac{m_d}{m_s}}$
  -- Fritzsch textures

- well-compatible with the see-saw and very stable versus M

$$\bar{5}\,\frac{w}{M}\,\bar{5}\,\Phi_5\Phi_5 \qquad \text{from} \qquad 1\,y_\nu\,\bar{5}\,\Phi_5 + 1\,M\,1$$

assuming

$$y_\nu \approx y_u \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow m_\nu = y_\nu^T M^{-1} y_\nu\, v_u^2 \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \frac{v_u^2}{M_{33}}$$

whatever M is!
$[M_{33} \neq 0]$

- LO picture can be translated into a more realistic model by replacing the zeros with small quantities
  $U(1)_{FN}$ abelian flavour symmetry
  spontaneously broken by $\lambda=\langle\vartheta\rangle/\Lambda < 1$
  -- fix mass relations of 1st and 2nd generation
  -- address DT splitting problem
  -- check gauge coupling unification, p-decay,…

[Altarelli,F 9812475; Altarelli, F, Masina 0007254]

[MSW SA/LA, LOW, VO]

# flavor puzzle made simpler in SU(5) ?

suppose that $y_u$, $y_e$, $y_\nu$ and $M/\Lambda$ are anarchical matrices [O(1) matrix elements] and that the observed hierarchy is due to some sort of wave function renormalization of matter multiplets

$$10 \rightarrow F_{10}\, 10 \qquad \bar{5} \rightarrow F_{\bar{5}}\, \bar{5}$$

$$F_X = \begin{pmatrix} \varepsilon'_X & 0 & 0 \\ 0 & \varepsilon_X & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$1 \geq \varepsilon_X \geq \varepsilon'_X$$

$F_X$ can arise from $U(1)_{FN}$ symmetries, a 5th Extra Dimension, Partial Compositness

large mixing in lepton sector suggests

$$F_{\bar{5}} \approx \mathrm{diag}(\varepsilon'_5, 1, 1)$$

hierarchy mostly due to $F_{10}$

$$F_{10} \approx \mathrm{diag}(\varepsilon'_{10}, \varepsilon_{10}, 1)$$

$$\Upsilon_u = F_{10} y_u F_{10} \qquad \Upsilon_d = F_{\bar{5}} y_d F_{10} \qquad \Upsilon_e = F_{10} y_d^T F_{\bar{5}}$$

in the extreme case $\varepsilon'_5 = 1$ [ANARCHY] [Hall, Murayama, Weiner 1999 De Gouvea, Murayama 1204.1249]

$$\Rightarrow \quad m_u : m_c : m_t \approx m_d^2 : m_s^2 : m_b^2 \approx m_e^2 : m_\mu^2 : m_\tau^2$$

$$V_{ub} \approx V_{us} \times V_{cb}$$

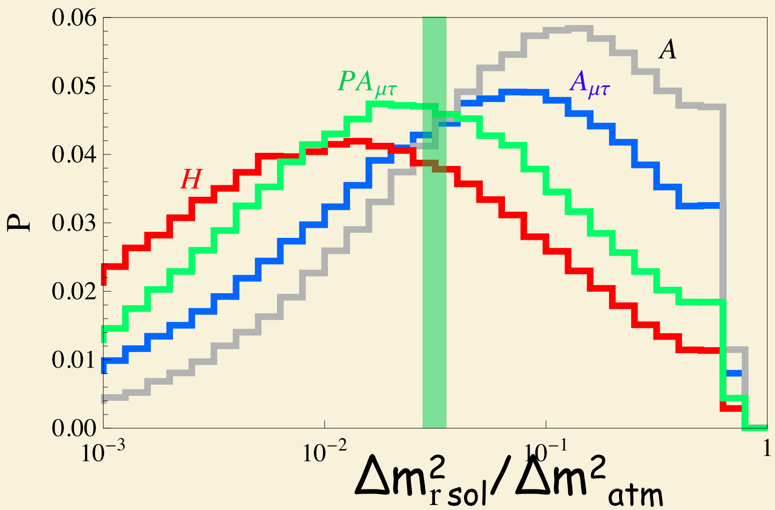
approximately true

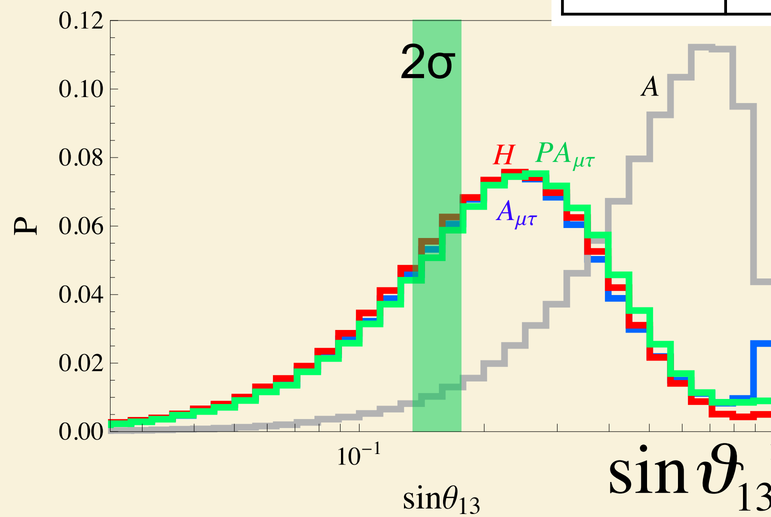
# but Guido was not an extremist!

| | $FN(\bar{5})$ | $\lambda$ |
|---|---|---|
| $A$ | (0,0,0) | |
| $A_{\mu\tau}$ | (1,0,0) | 0.25 |
| $PA_{\mu\tau}$ | (2,0,0) | 0.35 |
| $H$ | (2,1,0) | 0.45 |

$$F(\bar{5}_i) = \lambda^{FN(\bar{5}_i)}$$

[Buchmuller, Domcke, Schmitz, 1111.387;
Altarelli,F,Masina, Merlo 1207.0587;
Bergstrom, Meloni, Merlo, 1403.4528]

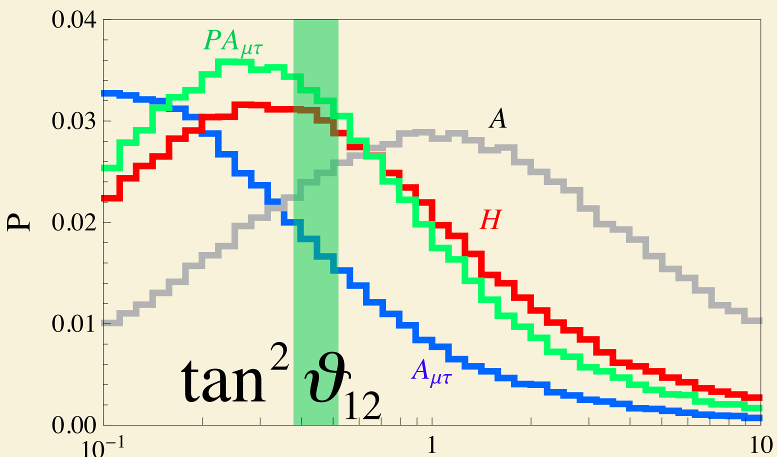


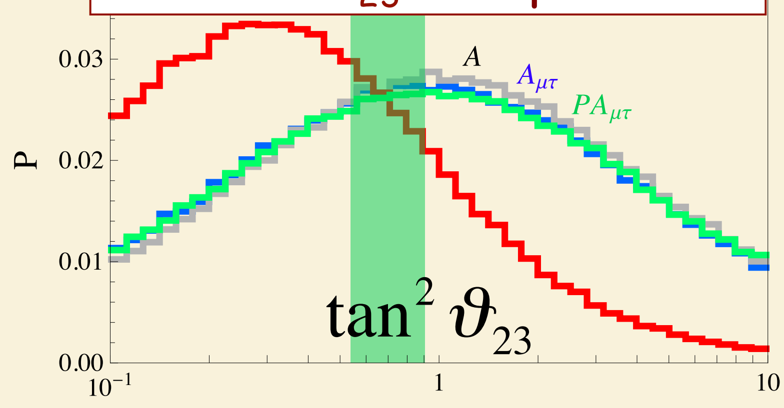

$$\sin^2\vartheta_{13} \approx \frac{\Delta m^2_{12}}{\Delta m^2_{13}}$$

NH favoured

large number of independent O(1) parameters

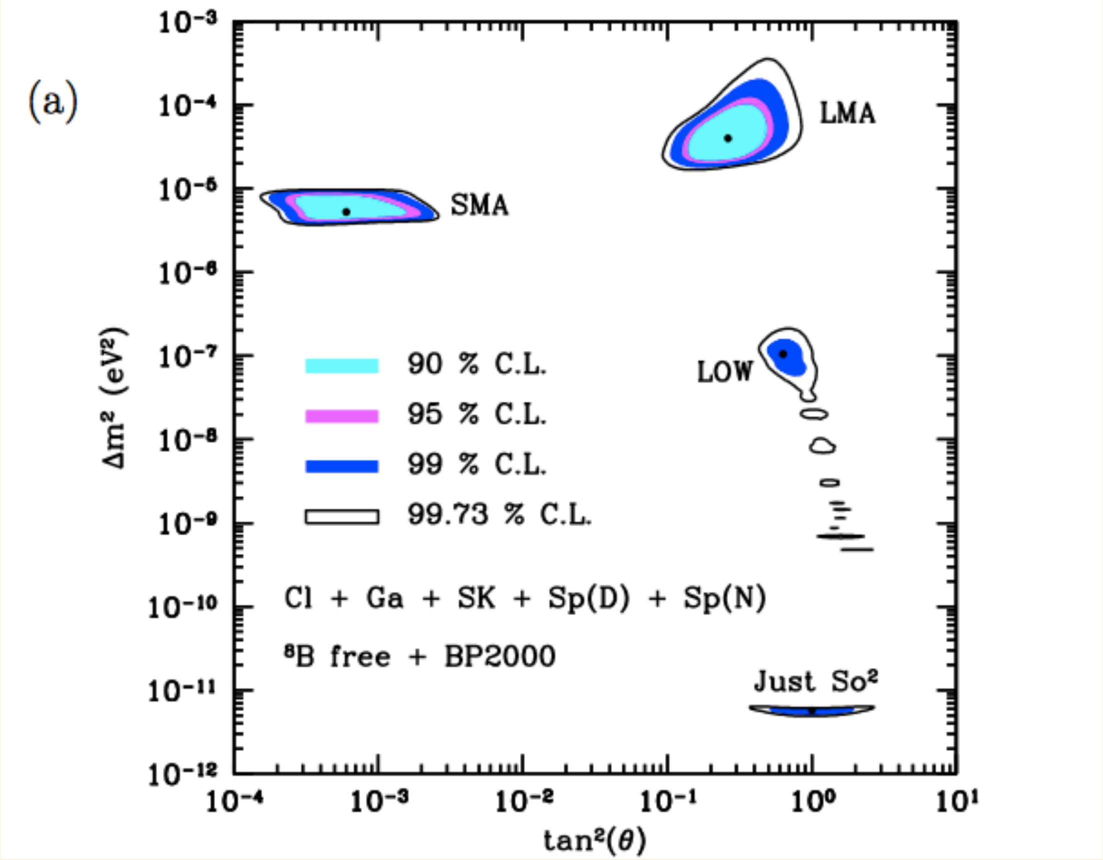



maximal $\vartheta_{23}$ unexplained




difficult to go beyond order-of-magnitude predictions

# Solar Neutrino Solutions < 2002




[Bahcall, Krastev, Smirnov 2001]

# 2002: the solar ν problem is solved

- by 2002 the MSW SA solution was ruled out by the large SK statistics [E-spectrum, time variation]
- **Direct Evidence for Neutrino Flavor Transformation from Neutral-Current Interactions in the Sudbury Neutrino Observatory**

  (Dated: 19 April 2002)

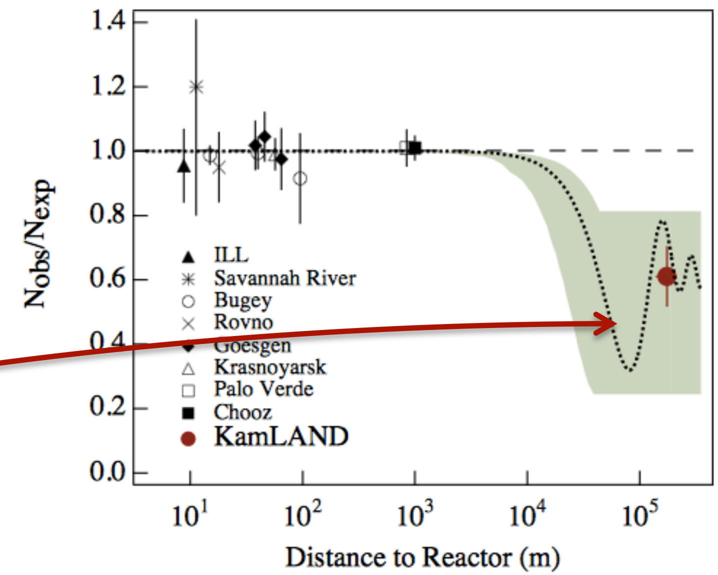
$$\begin{aligned} \nu_e + d &\rightarrow p + p + e^- &\quad (CC),\\ \nu_x + d &\rightarrow p + n + \nu_x &\quad (NC),\\ \nu_x + e^- &\rightarrow \nu_x + e^- &\quad (ES). \end{aligned}$$

$$\begin{aligned} \phi_e &= 1.76^{+0.05}_{-0.05}(\text{stat.})^{+0.09}_{-0.09}\ (\text{syst.})\\ \phi_{\mu\tau} &= 3.41^{+0.45}_{-0.45}(\text{stat.})^{+0.48}_{-0.45}\ (\text{syst.}) \end{aligned}$$

[MSW LA solution favoured, maximal $\vartheta_{12}$ mixing excluded]

- **First Results from KamLAND: Evidence for Reactor Anti-Neutrino Disappearance**

  (Dated: December 9, 2002)




KamLAND experiment exploits the low-energy electron anti-neutrinos (E≈3 MeV) produced by and Korean reactors at an average distance of L≈180 Km from the detector and is potentially Sensitive to $\Delta m^2$ down to $10^{-5}$ $eV^2$

MSW LA finally determined

$$\sin^2 2\theta = 0.833 \text{ and } \Delta m^2 = 5.5 \times 10^{-5}\ \text{eV}^2$$

# Tri-BiMaximal Mixing [TBM]

- MSW LA ➡ $\sin^2 \vartheta_{12} = 0.32^{+0.05}_{-0.06}$ [Bahcall, Gonzalez-Garcia, Pena-Garay 0212147]

[Harrison, Perkins, Scott 0202074]

$$(|U_{l\nu}|^2) = \begin{array}{c} \\ e \\ \mu \\ \tau \end{array} \begin{array}{c} \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\ \left( \begin{array}{ccc} 2/3 & 1/3 & 0 \\ 1/6 & 1/3 & 1/2 \\ 1/6 & 1/3 & 1/2 \end{array} \right) \end{array}$$

assuming $\vartheta_{13}$ negligible

so "symmetric" and soon derived from $A_4$ discrete symmetry

Ma, Rajasekaran 0106291, Babu, Ma, Valle 0206292; Hirsch, Romao, Skadauge, Valle, Villanova del Moral 0312244, Ma 0404199, 0409075]

- $A_4$ was the upgrade of the μ−τ parity symmetry [Grimus, Lavoura 0110041, 0305046] in the flavour basis, require $m_\nu$ invariant under U

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad U^2=1$$

$$m_\nu = \begin{pmatrix} x & y & y \\ y & w & z \\ y & z & w \end{pmatrix} \Leftrightarrow \begin{array}{l} \vartheta_{13} = 0 \\ \vartheta_{23} = \dfrac{\pi}{4} \end{array}$$

$\vartheta_{12}$ undetermined

TBM is obtained when $x + y = w + z$

now $m_\nu$ invariant also under S

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

$$U^2 = S^2 = 1 \quad [S,U] = 0$$

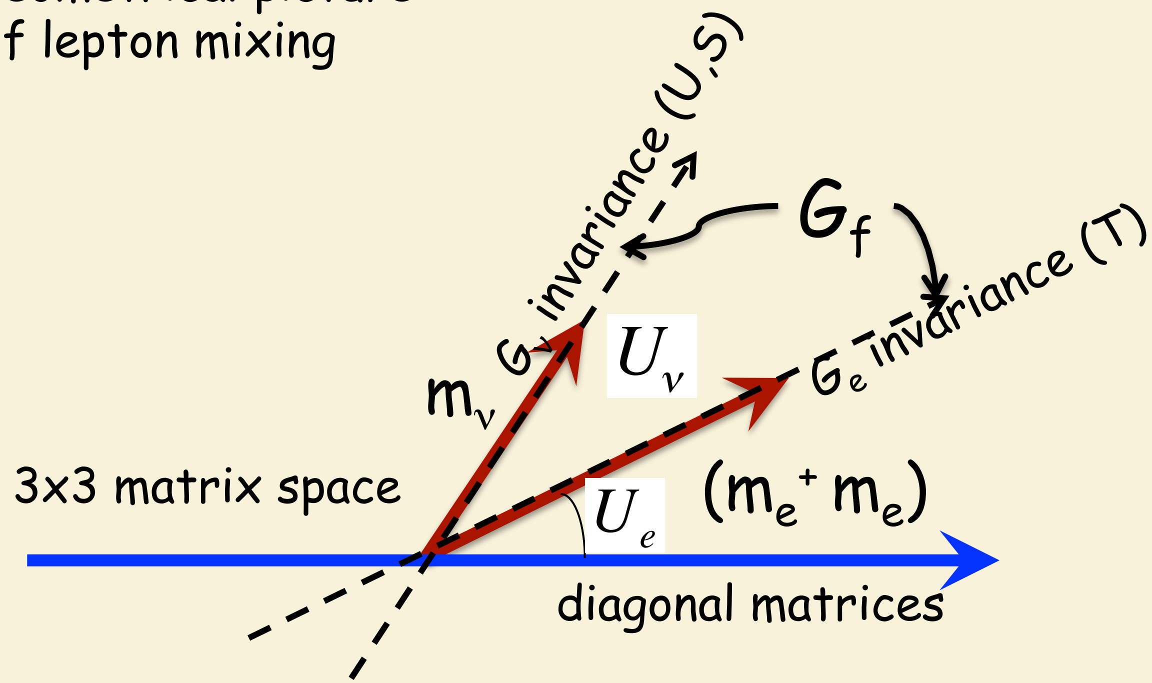
$Z_2 \times Z_2$ the most general symmetry of $m_\nu$ if neutrinos are Majorana the flavour basis can be guaranteed if $(m_e^+ m_e)$ is invariant under

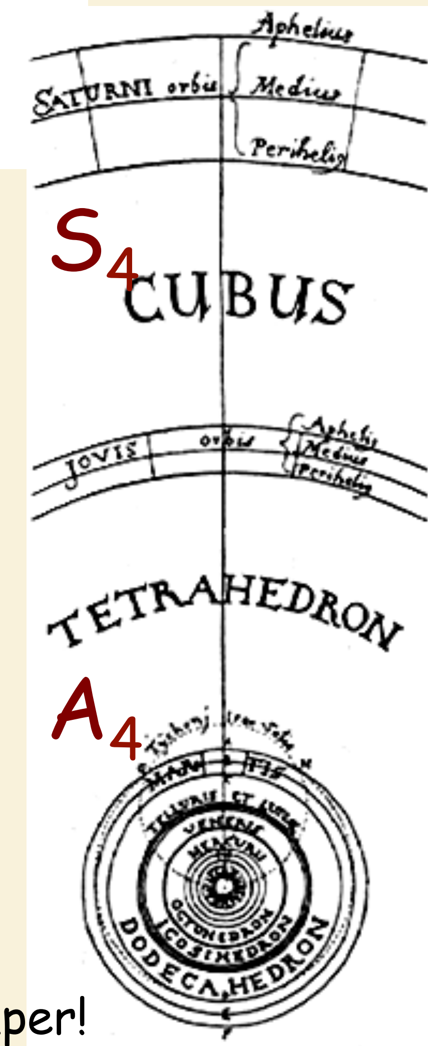
$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \qquad \omega = e^{i\frac{2\pi}{3}}$$

[Lam 0708.3665 + 0804.2622]

- (S,T) generate $A_4$ (U can arise as an accidental symmetry)
  (S,T,U) generate $S_4$
  ...
  ...

geometrical picture of lepton mixing




[Kepler 1596 Mysterium Cosmographicum]

very unfortunate Kepler's paper!

# Tri-BiMaximal Mixing from $A_4$ [AF 0504165, 0512103]

we built a model with a number of nice features...

- desired breaking – $G_\nu = \{U,S\}$ $G_e = \{T\}$ – achieved dynamically
  $G_\nu$ and $G_e$ selected by the minimum of the energy density of the theory

  vacuum alignment at LO $\langle \varphi_T \rangle = (1\,0\,0)\, V_T$ $\langle \varphi_S \rangle = (1\,1\,1)\, V_S$

- LO lepton mixing angles – TBM – completely determined by the breaking
  -- no ad-hoc relations among parameters required
  -- formalism totally basis independent

- μ–τ parity symmetry naturally incorporated: U generator arises as an accidental symmetry

- charged lepton mass hierarchy explained by $U(1)_{FN}$ (-> $Z_4$ in a more minimal version) [Altarelli, Meloni 0905.0620]

- study of NLO corrections induced by higher-dimensional operators,...

$$U_{PMNS} = U_{TB} + O(\varepsilon)$$

$$\varepsilon = \frac{V_T}{\Lambda}, \frac{V_S}{\Lambda}$$

expected size of ε fixed by the agreement $\vartheta_{12}^{TB} \approx \vartheta_{12}^{EXP}$ ➡ $0.01 < \varepsilon < 0.05$

and some alarming predictions…

$\vartheta_{23}$ nearly maximal — still compatible with data

$\vartheta_{13} < 0.05$ — wrong!

me: very much excited about this neat prediction!

Guido:

" Special models are those where some symmetry or dynamical feature assures in a natural way the near vanishing of $\theta_{13}$ and/or of $\theta_{23} - \pi/4$. Normal models are conceptually more economical and much simpler to construct. We expect that experiment will eventually find that $\theta_{13}$ is not too small and that $\theta_{23}$ is sizably not maximal." [Altarelli, 2005]

# 2011/2012 breakthrough: $\vartheta_{13} \neq 0$

- from LBL experiments searching for $\nu_\mu \to \nu_e$ conversion

T2K: muon neutrino beam produced at JPARC [Tokai] E=0.6 GeV and sent to SK 295 Km apart [1106.2822]

MINOS: muon neutrino beam produced at Fermilab [E=3 GeV] sent to Soudan Lab 735 Km apart [1108.0015]

$$P(\nu_\mu \to \nu_e) = \sin^2\vartheta_{23}\sin^2 2\vartheta_{13}\sin^2\frac{\Delta m^2_{32}L}{4E} + \ldots$$

both experiments favor $\sin^2\vartheta_{13} \sim$ few %

- from SBL reactor experiments searching for anti-$\nu_e$ disappearance

Double Chooz (far detector):

Daya Bay (near + far detectors):

RENO (near + far detectors):

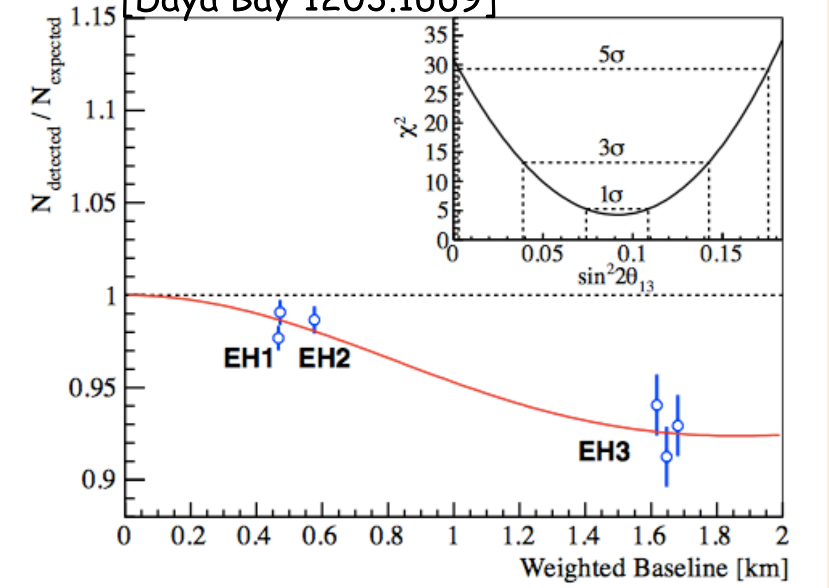
$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\vartheta_{13}\sin^2\frac{\Delta m^2_{32}L}{4E} + \ldots$$

DC: $\sin^2\vartheta_{13} = 0.022 \pm 0.013$

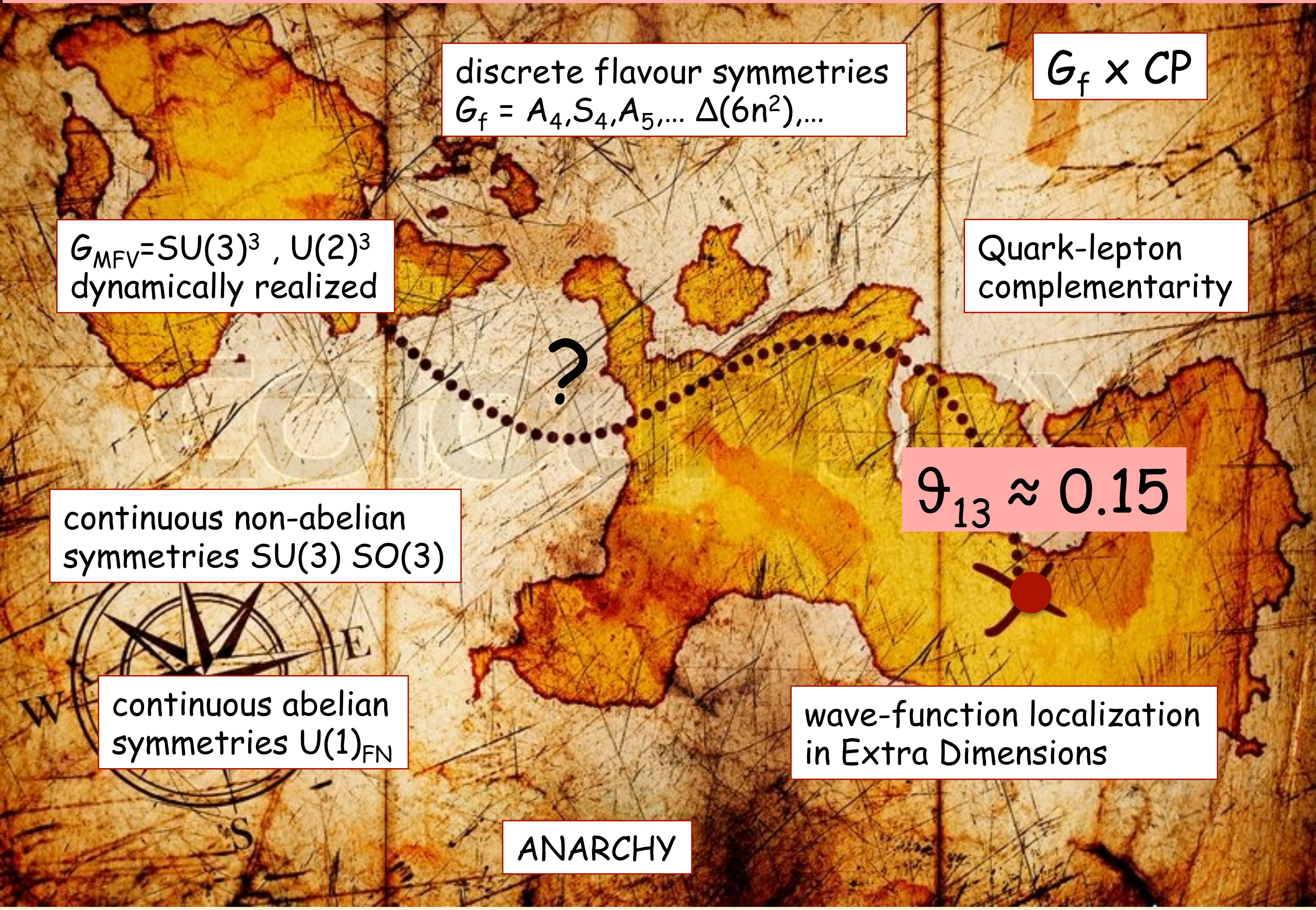
DB: $\sin^2\vartheta_{13} = 0.024 \pm 0.004$

R: $\sin^2\vartheta_{13} = 0.029 \pm 0.006$

[Daya Bay 1203.1669]



# Which Direction ?



Unfortunately $\vartheta_{13} \approx 0.15$ does not indicate any precise direction in the chart of possible models

$\vartheta_{13} \approx 0.15$ rad and the hint for non maximal $\vartheta_{23}$ have strengthened the case for anarchy, and for variants based on $U(1)_{FN}$ abelian continuous symmetries, Extra Dimensions,…

But discrete symmetries can also easily cope with $\vartheta_{13} \approx 0.15$

-- add "large" corrections $O(\vartheta_{13}) \approx 0.15$ to TBM pattern

-- change discrete group $G_f$ and try to fit lepton mixing

F.F., C. Hagedorn, R. de A.Toroop hep-ph/1107.3486 and hep-ph/1112.1340
Lam 1208.5527 and 1301.1736
Holthausen1, Lim and Lindner 1212.2411
Neder, King, Stuart 1305.3200
Hagedorn, Meroni, Vitale 1307.5308]

| $n$ | $G$ | GAP-Id | $\sin^2(\theta_{12})$ | $\sin^2(\theta_{13})$ | $\sin^2(\theta_{23})$ |
|---|---|---|---|---|---|
| 5 | $\Delta(6 \cdot 10^2)$ | [600, 179] | 0.3432 | 0.0288 | 0.3791 |
| | | | 0.3432 | 0.0288 | 0.6209 |

complete classification of $|U_{PMNS}|$ from any finite group available now!

[Fonseca, Grimus 1405.3678]

-- change LO pattern

$$U^0_{PMNS} = U_{BM}$$

[G. Altarelli, F.F., L. Merlo and E. Stamou hep-ph/1205.4670; Altarelli, Machado, Meloni 1504.05514]

-- include CP in the SB pattern

[F. F, C. Hagedorn and R. Ziegler 1211.5560, 1303.7178 Ding,King,Luhn,Stuart 1303.6180 Ding, King, Stuart 1307.4212]

-- relax symmetry requirements

[He, Zee 2007 and 2011, Grimus, Lavoura 2008, Grimus, Lavoura, Singraber 2009, Albright, Rodejohann 2009, Antusch, King, Luhn, Spinrath 2011, King, Luhn 2011, Hernandez,Smirnov 1204.0445]

[Guido, Corfu 2014]

The main problem of discrete flavour groups is not so much that $\theta_{13}$ is large but that there is no hint from quarks for them

- no clear role in the quark sector
  large hierarchies and small mixing angles seem not require discrete groups

- extension to GUTs possible (many existence proofs) but rather complicated
  quark mass ratios and quark mixing angles from small parameters $\neq \varepsilon$
  [$U(1)_{FN}$ , Extra Dimensions,...]

one could have imagined that neutrinos would bring a decisive boost towards the formulation of a comprehensive understanding of fermion masses and mixings. In reality it is frustrating that no real illumination was sparked on the problem of flavor. We can reproduce in many different ways the observations, in a wide range that goes from anarchy to discrete flavor symmetries but we have not yet been able to single out a unique and convincing baseline for the understanding of fermion masses and mixings. In spite of many interesting ideas and the formulation of many elegant models the mysteries of the flavor structure of the three generations of fermions have not been much unveiled.

[Guido Altarelli, "Status of Neutrino Mass and Mixing" 1404.3859]

# Conclusion

From the theoretical side, for $\nu$ masses and mixings
we do not have so far a compelling theoretical picture
and many possibilities are still open.

Actually, also for quarks and charged leptons
we do not have a theory of flavour that explains the observed
spectrum, mixings and CP violation.

Yet in spite of impressive progress important
experimental open questions remain:
Absolute scale of $m^2$? Inverse or normal hierarchy?
CP violation? Flavour symmetry? Sterile $\nu$'s? DM?..

Thus $\nu$'s are interesting because they can provide new clues
on the flavour problem

[Guido, Corfu 2014]


I will miss you a lot, Guido !

1st Guido paper on neutrino masses

# NEUTRINO MASSES: A THEORETICAL INTRODUCTION

**Guido Altarelli**
CERN - Geneva

**Content**



*Invited talk given at the 6th International Symposium on "Neutrino Telescopes"*
*Venice, Italy, February 1994*

# Backup slides

# anything special from data, requiring a symmetry?

1. $\vartheta_{23}$ maximal ?
2. $\delta_{CP} = -\pi/2$ ?
3. $U_{PMNS}$ close to TB (BM,…) ?

3 examples from a longer list…

1 today most precise single determination of $\vartheta_{23}$ is from T2K ($P_{\mu\mu}$)

[1403.1532]

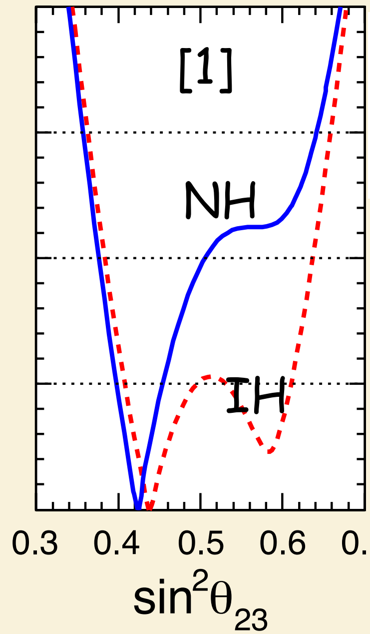
$$\sin^2\vartheta_{23} = \begin{cases} 0.514^{+0.055}_{-0.056} & \text{(NH)} \\ 0.511^{+0.055}_{-0.055} & \text{(IH)} \end{cases}$$

well compatible with $\vartheta_{23}$ maximal

global fits hint at $\vartheta_{23}$ non-maximal
main effect: interplay between SBL reactor experiments ($P_{ee}$) and LBL experiments searching ($P_{\mu e}$)

$$P_{ee} = 1 - \sin^2 2\vartheta_{13} \sin^2\frac{\Delta m^2_{32} L}{4E} + \ldots$$

$$P_{\mu e} = \sin^2\vartheta_{23} \sin^2 2\vartheta_{13} \sin^2\frac{\Delta m^2_{32} L}{4E} + \ldots$$




$$\sin^2\vartheta_{23} = \begin{cases} 0.567^{+0.032}_{-0.128} & \text{(NH)} \\ 0.573^{+0.025}_{-0.043} & \text{(IH)} \end{cases}$$ [2]

global fit:
[1] Capozzi, Fogli, Lisi, Marrone, Montanino, Palazzo 1312.2878
[2] Forero, Tortola, Valle 1405.7540

a small change of $P_{ee}$ and/or $P_{\mu e}$ within about $1\sigma$ can bring back $\vartheta_{23}$ to maximal difficult to improve $\vartheta_{23}$ from $P_{\mu\mu}$

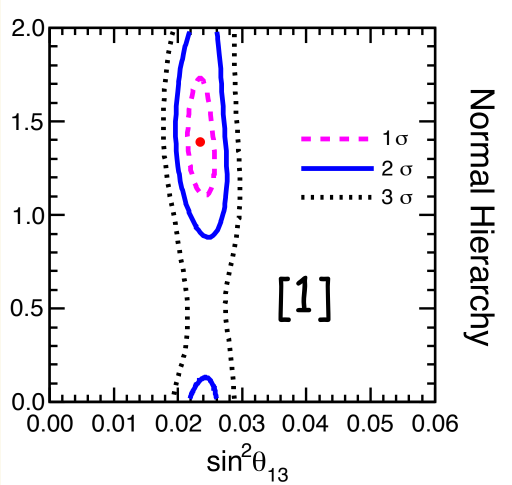
$$\delta\vartheta_{23} \approx \sqrt{\delta P_{\mu\mu}} / 2 \qquad \delta P_{\mu\mu} \approx 0.01 \quad \Rightarrow \quad \delta\vartheta_{23} \approx 0.05 \text{ rad } (2.9^0)$$

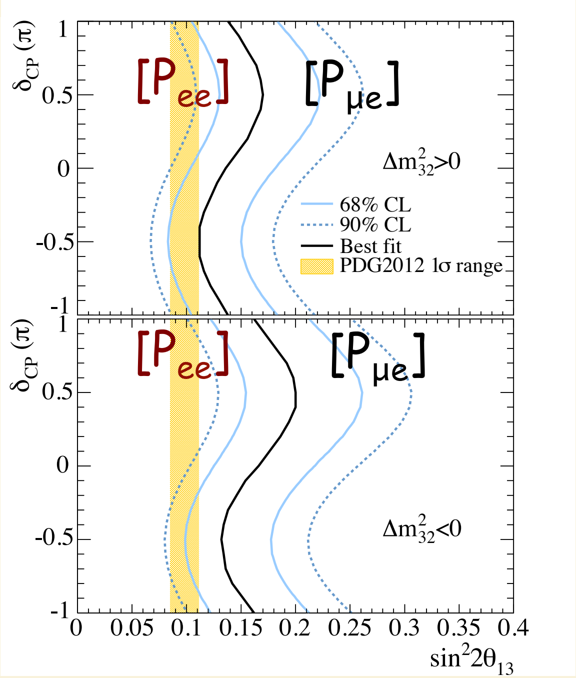
$\vartheta_{23}$ nearly maximal would be a crucial piece of information

- $\vartheta_{23}$ cannot be made maximal by RGE evolution [barring tuning of b.c. and/or thresold corrections]
- when a flavour symmetry is present, $\vartheta_{23}$ is determined entirely by breaking effects [no maximal $\vartheta_{23}$ from an exact symmetry]

broken abelian symmetries do not work [not a theorem but no counterexamples] → we are left with broken non-abelian symmetries

**2** $\delta_{CP} = -\pi/2$ ?




[T2K: 1311.4750 and 1311.4114]

## 1 add large corrections $O(\vartheta_{13}) \approx 0.2$

- predictability is lost since in general correction terms are many
- new dangerous sources of FC/CPV if NP is at the TeV scale

## 2 relax symmetry requirements

[Hernandez,Smirnov 1204.0445]

$G_e$ as before

$G_\nu = Z_2$

2 predictions:
2 combinations of $\vartheta^0_{12}$ $\vartheta^0_{23}$ $\vartheta^0_{13}$ $\delta_{CP}$

two deformations of TB, called Trimaximal [TM] mixing

$TM_1$

$$U^0 = U_{TB} \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & e^{i\delta}\sin\alpha \\ 0 & -e^{-i\delta}\sin\alpha & \cos\alpha \end{pmatrix}$$

$TM_2$

$$U^0 = U_{TB} \times \begin{pmatrix} \cos\alpha & 0 & e^{i\delta}\sin\alpha \\ 0 & 1 & 0 \\ -e^{-i\delta}\sin\alpha & 0 & \cos\alpha \end{pmatrix}$$

leads to testable sum rules

$$\sin^2\vartheta_{12} = \frac{1}{3} - \frac{2}{3}\sin^2\vartheta_{13} + O(\sin^4\vartheta_{13})$$

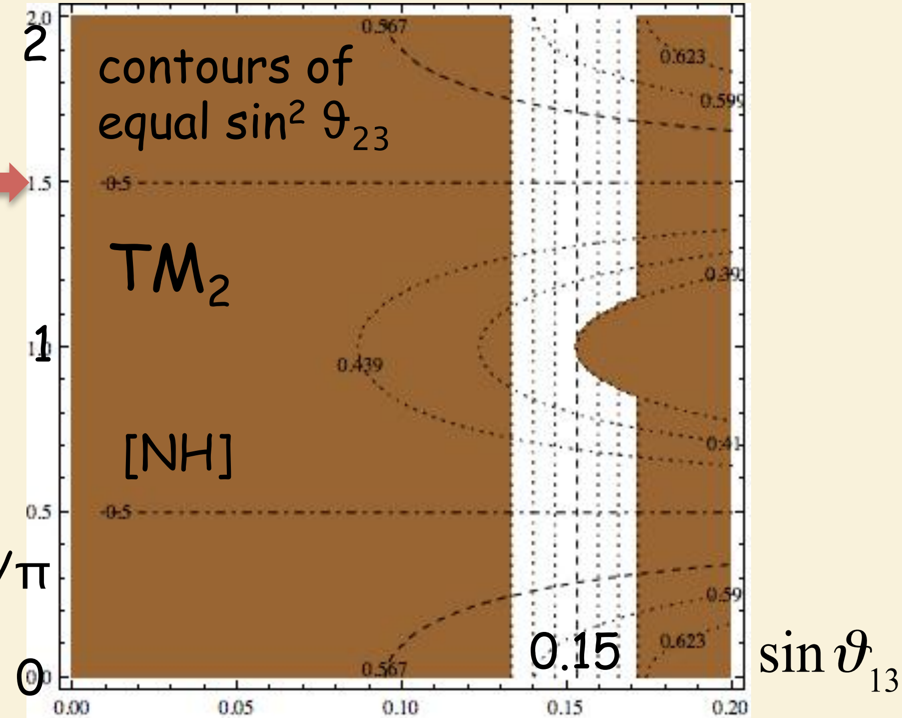
$$\sin^2\vartheta_{23} = \frac{1}{2} - \sqrt{2}\sin\vartheta_{13}\cos\delta_{CP} + O(\sin^2\vartheta_{13})$$

$$\sin^2\vartheta_{12} = \frac{1}{3} + \frac{1}{3}\sin^2\vartheta_{13} + O(\sin^4\vartheta_{13})$$

$$\sin^2\vartheta_{23} = \frac{1}{2} + \frac{1}{\sqrt{2}}\sin\vartheta_{13}\cos\delta_{CP} + O(\sin^2\vartheta_{13})$$

[He, Zee 2007 and 2011, Grimus, Lavoura 2008, Grimus, Lavoura, Singraber 2009, Albright, Rodejohann 2009, Antusch, King, Luhn, Spinrath 2011, King, Luhn 2011, G. Altarelli, F.F., L. Merlo and E. Stamou hep-ph/1205.4670 ] deviation from TB is linear in α for $\sin^2\theta_{23}$, whereas is quadratic for $\sin^2\theta_{12}$, the best measured angle

sum rules can be tested by measuring $\delta_{CP}$ and improving on $\sin^2 \vartheta_{23}$




3 change discrete group $G_f$

- solutions exist
  special forms of TM$_2$

| $G_f$ | $\Delta(96)$ | $\Delta(384)$ | $\Delta(600)$ |
|---|---|---|---|
| $\alpha$ | $\pm\pi/12$ | $\pm\pi/24$ | $\pm\pi/15$ |
| $\sin^2\vartheta_{13}^0$ | 0.045 | 0.011 | 0.029 |

$$U^0 = U_{TB} \times \begin{pmatrix} \cos\alpha & 0 & e^{i\delta}\sin\alpha \\ 0 & 1 & 0 \\ -e^{-i\delta}\sin\alpha & 0 & \cos\alpha \end{pmatrix}$$

$\delta^0 = 0, \pi$ (no CP violation) and
α "quantized" by group theory

complete classification of $|U_{PMNS}|$ from any finite group available now!

F.F., C. Hagedorn, R. de A.Toroop
hep-ph/1107.3486 and hep-ph/1112.1340
Lam 1208.5527 and 1301.1736
Holthausen1, Lim and Lindner 1212.2411
Neder, King, Stuart 1305.3200
Hagedorn, Meroni, Vitale 1307.5308]

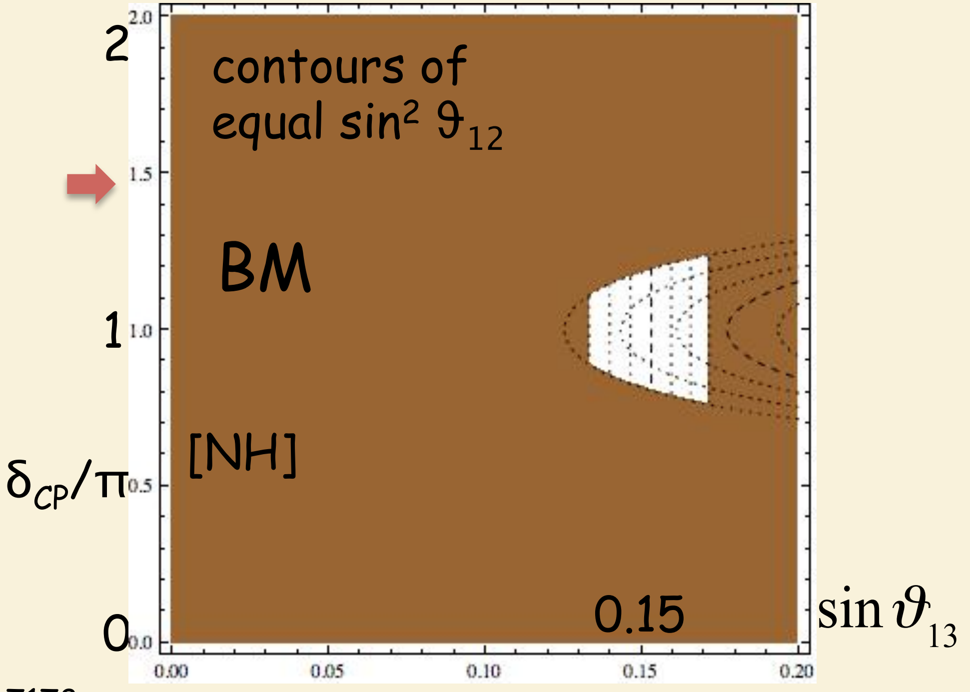
[Fonseca, Grimus 1405.3678]

4 change LO pattern

$$U^0_{PMNS} = U_{BM}$$

corrected by $U^e_{12}$

$$\sin^2\vartheta_{12} = \frac{1}{2} + \sin\vartheta_{13}\cos\delta_{CP} + O(\sin^2\vartheta_{13})$$




5 include CP in the SB pattern

$$G_{CP} = G_f \rtimes CP$$

[F. F, C. Hagedorn and R. Ziegler 1211.5560, 1303.7178 Ding,King,Luhn,Stuart 1303.6180 Ding, King, Stuart 1307.4212]

$$G_e \qquad G_\nu = Z_2 \times CP$$

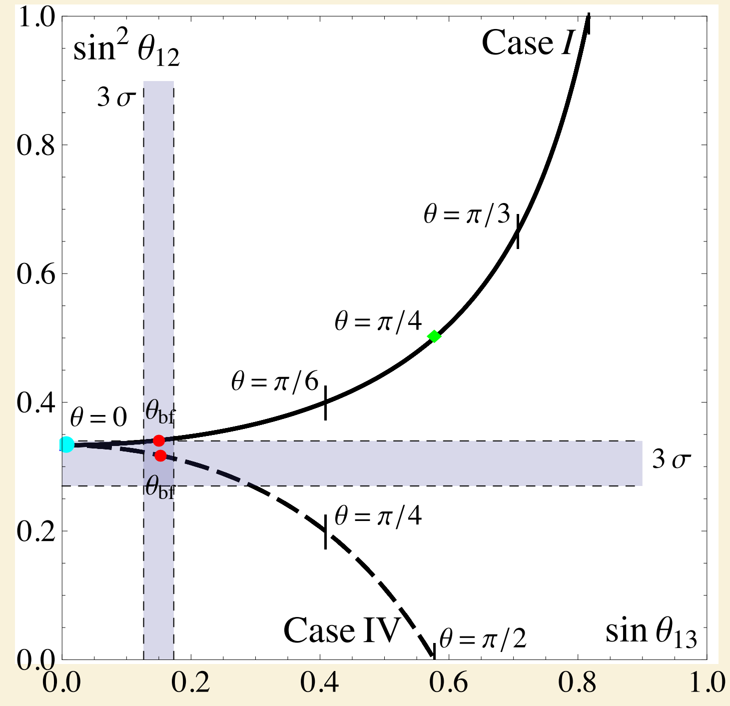
mixing angles and CP violating phases

$$(\vartheta^0_{12}, \vartheta^0_{23}, \vartheta^0_{13}, \delta^0, \alpha^0, \beta^0)$$

predicted in terms of a single real parameter $0 \leq \vartheta \leq \pi$

2 examples with $G_f=S_4$ $G_e=Z_3$

$$\sin^2\vartheta^0_{23} = \frac{1}{2} \qquad |\sin\delta^0| = 1 \qquad \sin\alpha^0 = 0 \qquad \sin\beta^0 = 0$$



# 2011/2012 breakthrough

-- LBL experiments searching for $\nu_\mu \to \nu_e$ conversion [see Fogli's talk]
-- SBL reactor experiments searching for anti-$\nu_e$ disappearance

| | Lisi [NeuTel 2013] | [1209.3023] [G-Garcia, Maltoni, Salvado, Schwetz] |
|---|---|---|
| $\sin^2\vartheta_{13}$ | $0.0241^{+0.0025}_{-0.0025}\ (NO)$ <br> $0.0244^{+0.0023}_{-0.0025}\ (IO)$ | $0.0227^{+0.0023}_{-0.0024}$ |
| $\sin^2\vartheta_{23}$ | $0.386^{+0.024}_{-0.021}\ (NO)$ <br> $0.392^{+0.039}_{-0.022}\ (IO)$ | $0.413^{+0.037}_{-0.025} \oplus 0.594^{+0.021}_{-0.022}$ |

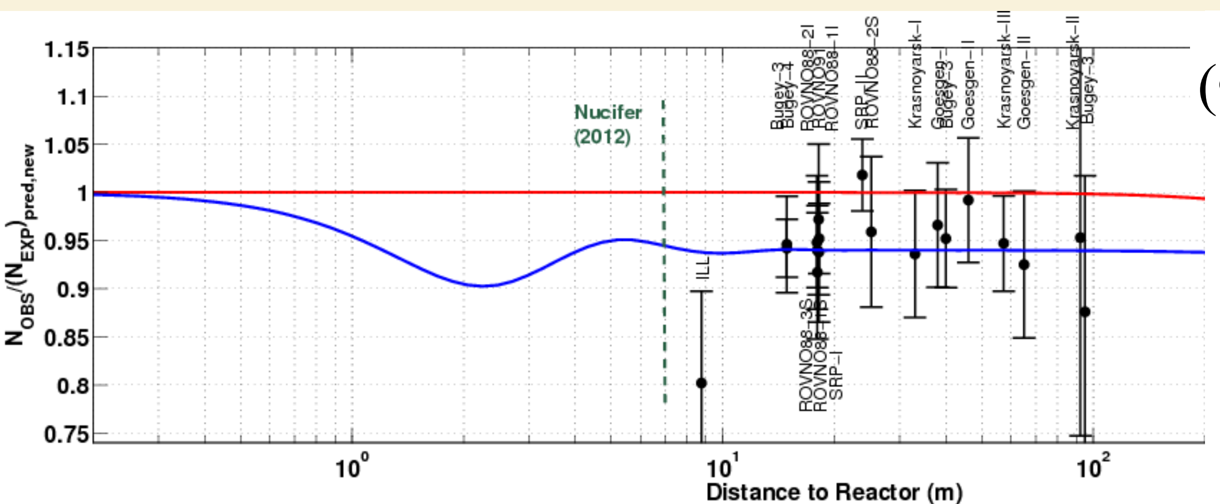
→ 10σ away from 0 — impact on flavor symmetry (part 3)

→ hint for non maximal $\vartheta_{23}$

# sterile neutrinos coming back

1 reactor anomaly (anti-$\nu_e$ disappearance)

re-evaluation of reactor anti-$\nu_e$ flux: new estimate 3.5% higher than old one




$$(\Phi_{exp} - \Phi_{th})/\Phi_{th} \approx -6\%$$

[th. uncertainty?]

very SBL L ≲ 100 m

$$\vartheta_{es} \approx 0.2$$

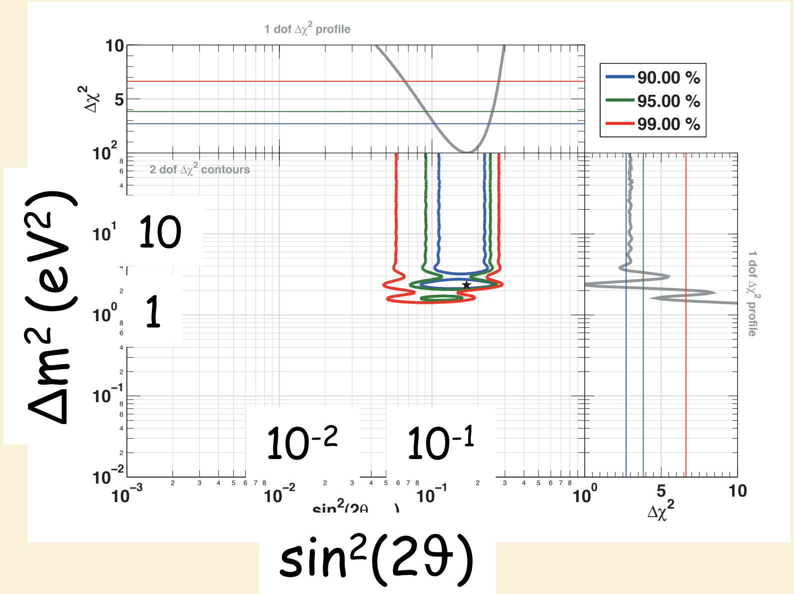
$$\Delta m^2 \approx m_s^2 \geq 1\, eV^2$$

supported by the Gallium anomaly

$\nu_e$ flux measured from high intensity radioactive sources in Gallex, Sage exp

$$\nu_e + {}^{71}Ga \rightarrow {}^{71}Ge + e^-$$

[error on σ or on Ge extraction efficiency]




## most recent cosmological limits

[depending on assumed cosmological model, data set included,…]

relativistic degrees of freedom at recombination epoch

$$N_{eff} = 3.30 \pm 0.27$$

[Planck, WMAP, BAO, high multiple CMB data]

fully thermalized non relativistic ν

$$N_{eff} < 3.80 \quad (95\%\ CL)$$

$$m_s < 0.42\ eV \quad (95\%\ CL)$$

## 2 long-standing claim

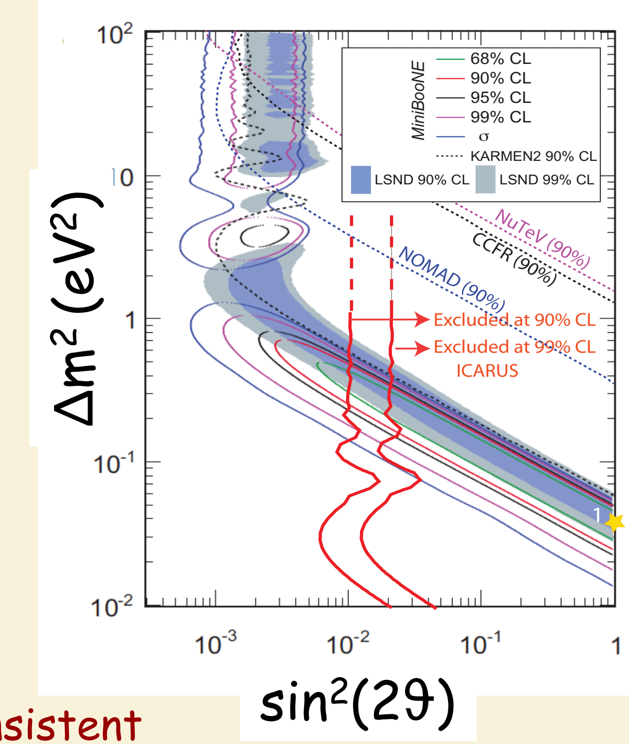
evidence for $\nu_\mu \rightarrow \nu_e$ appearance in accelerator experiments

| exp | | $E(MeV)$ | $L(m)$ | |
|---|---|---|---|---|
| *LSND* | $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ | $10 \div 50$ | 30 | 3.8σ |
| *MiniBoone* | $\nu_\mu \rightarrow \nu_e$<br>$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ | $300 \div 3000$ | 541 | 3.8σ [signal from low-energy region] |

parameter space limited by negative results from Karmen and ICARUS

$$\vartheta_{e\mu} \approx 0.035$$

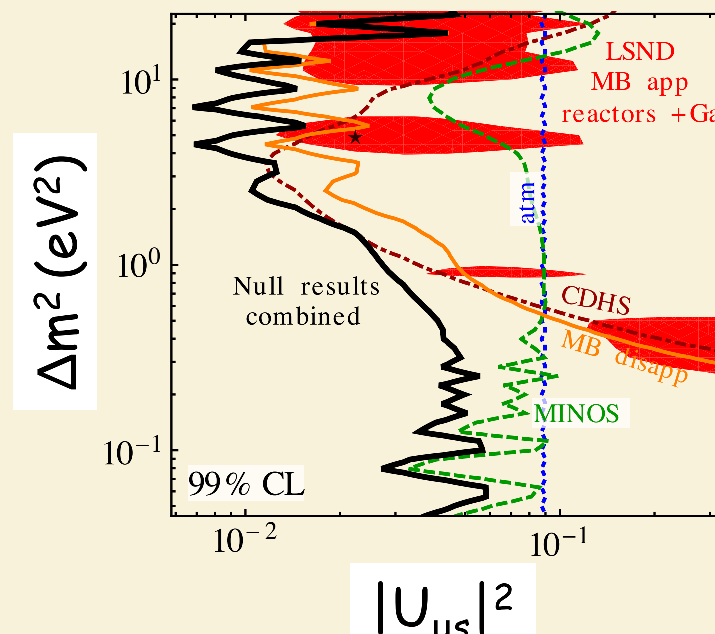
$$\Delta m^2 \approx 0.5\, eV^2$$




3 interpretation in 3+1 scheme: inconsistent (more than 1s disfavored by cosmology)

$$\underbrace{\vartheta_{e\mu}}_{0.035} \approx \underbrace{\vartheta_{es}}_{0.2} \times \vartheta_{\mu s} \Rightarrow \vartheta_{\mu s} \approx 0.2$$

predicted suppression in $\nu_\mu$ disappearance experiments: undetected




by ignoring LSND/Miniboone data the reactor anomaly can be accommodated by $m_s \geq 1$ eV and $\vartheta_{es} \approx 0.2$
[not suitable for WDM, more on this later]

# $A_4$ as a leftover of Poincare symmetry in D>4

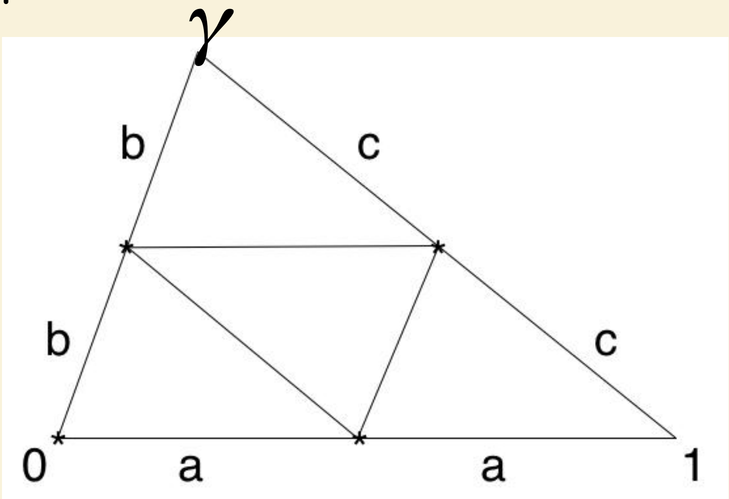
[AFL]

D dimensional
Poincare symmetry:
D-translations x SO(1,D-1)

⟺

usually broken by
compactification down to 4 dimensions:
4-translations x SO(1,3) x …

a discrete subgroup of the (D-4) euclidean group = translations x rotations
can survive in specific geometries




Example: D=6

2 dimensions
compactified on $T^2/Z_2$

$$z \to z+1$$
$$z \to z+\gamma$$
$$z \to -z$$

four fixed points

$$\text{if} \quad \gamma = e^{i\frac{\pi}{3}}$$

compact space is a regular tetrahedron
invariant under

$$S: \quad z \to z + \frac{1}{2}$$ [translation]

$$T: \quad z \to \gamma^2 z$$ [rotation by $120^0$]

[subgroup of 2 dim Euclidean group = 2-translations x SO(2)]